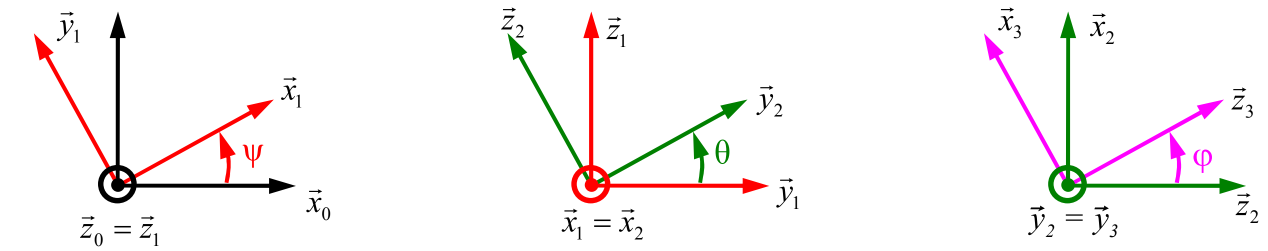
EXERCICE 13 CENTRIFUGEUSE HUMAINE

Figures de changements de base :



Q.1. Le plan $\left(O,\vec{y}_1,\vec{z}_1\right)$ est un plan de symétrie pour le sous ensemble 1 → $\left(O,\vec{x}_1\right)$ axe principal d'inertie → $\overline{\overline{I(G_1,S_1)}} = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_1 & -D_1 \\ 0 & -D_1 & C_1 \end{pmatrix}_{B_1}$

Q.2. 1/0 : Mouvement de rotation autour d'un axe fixe + matrice d'inertie donnée en G_1 (centre de gravité) :

$$\mathcal{C}_{1/0} = \underset{G_1}{\begin{Bmatrix} \overrightarrow{R_{C\ 1/0}} = m_1.\overrightarrow{V_{G_1,1/0}} \\ \overrightarrow{\sigma_{G_1,\ 1/0}} = \overline{\overline{I(G_1,S_1)}}.\overrightarrow{\Omega_{1/0}} \end{Bmatrix}} \quad \text{avec : } \overrightarrow{R_{C\ 1/0}} = m_1.\overrightarrow{V_{G_1,1/0}}$$

$$\overrightarrow{V_{G_1,1/0}} = \cancel{\overrightarrow{V_{O,1/0}}} + \overrightarrow{G_1O} \wedge \overrightarrow{\Omega_{1/0}} = -a.\vec{y}_1 \wedge \dot{\psi}.z_1 = -a.\dot{\psi}.\vec{x}_1 \Rightarrow \overrightarrow{R_{C\ 1/0}} = -m_1.a.\dot{\psi}.\vec{x}_1$$

$$\overrightarrow{\sigma_{G_1,\ 1/0}} = \overline{\overline{I(G_1,S_1)}}.\overrightarrow{\Omega_{1/0}} = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_1 & -D_1 \\ 0 & -D_1 & C_1 \end{pmatrix}_{B_1} . \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix}_{B_1} = \begin{pmatrix} 0 \\ -D_1.\dot{\psi} \\ C_1.\dot{\psi} \end{pmatrix}_{B_1}$$

On déplace le moment cinétique en O :

$$\overrightarrow{\sigma_{O,\ 1/0}} = \overrightarrow{\sigma_{G_1,\ 1/0}} + \overrightarrow{OG_1} \wedge \overrightarrow{R_{C\ 1/0}} = \overrightarrow{\sigma_{G_1,\ 1/0}} + a.\vec{y}_1 \wedge -m_1.a.\dot{\psi}.\vec{x}_1$$

$$\overrightarrow{\sigma_{O_1,\ 1/0}} = -D_1.\dot{\psi}.\vec{y}_1 + C_1.\dot{\psi}.\vec{z}_1 + m_1.a^2.\dot{\psi}.\vec{z}_1$$

$$\boxed{\mathcal{C}_{1/0} = \underset{O}{\begin{Bmatrix} -m_1.a.\dot{\psi}.\vec{x}_1 \\ -D_1.\dot{\psi}.\vec{y}_1 + (C_1 + m_1.a^2)\cdot\dot{\psi}.\vec{z}_1 \end{Bmatrix}}}$$

Q.3. Les plans $\left(I,\vec{x}_2,\vec{y}_2\right)$ et $\left(I,\vec{y}_2,\vec{z}_2\right)$ sont des plans de symétrie pour le solide 2 → la matrice est diagonale → $\overline{\overline{I(I,S_2)}} = \begin{pmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{pmatrix}_{B_2}$

Q.4. 2/0 : Mouvement quelconque + matrice d'inertie donnée en I (centre de gravité) :

$$\mathcal{C}_{2/0} = \left\{ \begin{array}{l} \overrightarrow{R_{C\ 2/0}} = m_2.\overrightarrow{V_{I,2/0}} \\ \overrightarrow{\sigma_{I,\ 2/0}} = \overline{\overline{I_I(S_2)}}.\overrightarrow{\Omega_{2/0}} \end{array} \right\}_I \text{ avec : } \overrightarrow{R_{C\ 2/0}} = m_2.\overrightarrow{V_{I,2/0}}$$

$$\overrightarrow{V_{I,2/0}} = \cancel{\overrightarrow{V_{I,2/1}}} + \overrightarrow{V_{I,1/0}} = \cancel{\overrightarrow{V_{O,1/0}}} + \overrightarrow{IO} \wedge \overrightarrow{\Omega_{1/0}} = R.\vec{y}_1 \wedge \dot{\psi}.z_1 = R.\dot{\psi}.\vec{x}_1 \Rightarrow \overrightarrow{R_{C\ 2/0}} = m_2.R.\dot{\psi}.\vec{x}_1$$

$$\overrightarrow{\sigma_{I,\ 2/0}} = \overline{\overline{I(I,S_2)}}.\overrightarrow{\Omega_{2/0}} = \begin{pmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{pmatrix}_{B_2} . \begin{pmatrix} \dot{\theta} \\ \dot{\psi}.sin\theta \\ \dot{\psi}.cos\theta \end{pmatrix}_{B_2} = \begin{pmatrix} A_2 \cdot \dot{\theta} \\ B_2.\dot{\psi}.sin\theta \\ C_2.\dot{\psi}.cos\theta \end{pmatrix}_{B_2}$$

	$\vec{y}_2$	$\vec{z}_2$
$\vec{y}_1$	$C\theta$	$-S\theta$
$\vec{z}_1$	$S\theta$	$C\theta$

$$\boxed{\mathcal{C}_{2/0} = \left\{ \begin{array}{c} m_2.R.\dot{\psi}.\vec{x}_1 \\ A_2.\dot{\theta}.\vec{x}_2 + B_2.\dot{\psi}.sin\theta.\vec{y}_2 + C_2.\dot{\psi}.cos\theta.\vec{z}_2 \end{array} \right\}_I}$$

Q.5. 3/0 : Mouvement quelconque + matrice d'inertie donnée en I (centre de gravité) :

$$\mathcal{C}_{3/0} = \left\{ \begin{array}{l} \overrightarrow{R_{C\ 3/0}} = m_3.\overrightarrow{V_{I,3/0}} \\ \overrightarrow{\sigma_{I,\ 3/0}} = \overline{\overline{I(I,S_3)}}.\overrightarrow{\Omega_{3/0}} \end{array} \right\}_I \text{ avec : } \overrightarrow{R_{C\ 3/0}} = m_3.\overrightarrow{V_{I,3/0}} = m_3.R.\dot{\psi}.\vec{x}_1$$

Car avec la pivot d'axe $(I,\vec{y}_2)$, $\overrightarrow{V_{I,3/0}} = \cancel{\overrightarrow{V_{I,3/2}}} + \overrightarrow{V_{I,2/0}}$

$$\overrightarrow{\sigma_{I,\ 3/0}} = \overline{\overline{I(I,S_3)}}.\overrightarrow{\Omega_{3/0}} = \begin{pmatrix} A_3 & 0 & 0 \\ 0 & B_3 & 0 \\ 0 & 0 & C_3 \end{pmatrix}_{B_2} . \begin{pmatrix} \dot{\theta} \\ \dot{\psi}.sin\theta + \dot{\varphi} \\ \dot{\psi}.cos\theta \end{pmatrix}_{B_2} = \begin{pmatrix} A_3 \cdot \dot{\theta} \\ B_3.(\dot{\psi}.sin\theta + \dot{\varphi}) \\ C_3.\dot{\psi}.cos\theta \end{pmatrix}_{B_2}$$

$$\boxed{\mathcal{C}_{3/0} = \left\{ \begin{array}{c} m_3.R.\dot{\psi}.\vec{x}_1 \\ A_3.\dot{\theta}.\vec{x}_2 + B_3.(\dot{\psi}.sin\theta + \dot{\varphi}).\vec{y}_2 + C_3.\dot{\psi}.cos\theta.\vec{z}_2 \end{array} \right\}_I}$$

Q.6. Au point I on a : $\left\{\mathcal{C}_{E1/0}\right\} = \left\{\mathcal{C}_{2/0}\right\} + \left\{\mathcal{C}_{3/0}\right\}$

$$\boxed{\mathcal{C}_{E_1/0} = \left\{ \begin{array}{c} (m_2 + m_3).R.\dot{\psi}.\vec{x}_1 \\ (A_2 + A_3).\dot{\theta}.\vec{x}_2 + \left[(B_2 + B_3).(\dot{\psi}.sin\theta) + B_3.\dot{\varphi})\right].\vec{y}_2 + (C_2 + C_3).\dot{\psi}.cos\theta.\vec{z}_2 \end{array} \right\}_I}$$

Q.7. $\mathcal{D}_{1/0} = \left\{ \begin{matrix} \overrightarrow{R_{d\ 1/0}} = m_1.\overrightarrow{\Gamma_{G_1,1/0}} \\ \overrightarrow{\delta_{O,\,1/0}} \end{matrix} \right\}_O$

Avec $\overrightarrow{\Gamma_{G_1,1/0}} = \left.\dfrac{d\overrightarrow{V_{G_1,1/0}}}{dt}\right|_{B0} = \left.\dfrac{d(a.\dot{\psi}.\vec{x}_1)}{dt}\right|_{B0} = -a.\ddot{\psi}.\vec{x}_1 - a.\dot{\psi}.(\dot{\psi}.\vec{z}_1 \wedge \vec{x}_1) = -a.\ddot{\psi}.\vec{x}_1 - a.\dot{\psi}^2.\vec{y}_1$

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O point fixe de 0 : $\overrightarrow{\delta_{O,\,1/0}} = \left.\dfrac{d\overrightarrow{\sigma_{O,1/0}}}{dt}\right|_{B0}$

$$\overrightarrow{\delta_{O,\,1/0}} = \begin{pmatrix} 0 \\ -D_1.\ddot{\psi} \\ (C_1 + m_1.a^2)\cdot\ddot{\psi} \end{pmatrix}_{B1} + \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix}_{B1} \wedge \begin{pmatrix} 0 \\ -D_1.\dot{\psi} \\ (C_1 + m_1.a^2)\cdot\dot{\psi} \end{pmatrix}_{B1} = \begin{pmatrix} D_1.\dot{\psi}^2 \\ -D_1.\ddot{\psi} \\ (C_1 + m_1.a^2)\cdot\ddot{\psi} \end{pmatrix}_{B1}$$

$$\boxed{\mathcal{D}_{1/0} = \left\{ \begin{matrix} m_1.a.\ddot{\psi}.\vec{x}_1 - m_1.a.\dot{\psi}^2.\vec{y}_1 \\ D_1.\dot{\psi}^2.\vec{x}_1 - D_1.\ddot{\psi}.\vec{y}_1 + (C_1 + m_1.a^2).\ddot{\psi}.\vec{z}_1 \end{matrix} \right\}_O}$$

Q.8. On décompose en sous-systèmes élémentaires :

$$\mathcal{D}_{E_2/0} = \mathcal{D}_{1/0} + \mathcal{D}_{2/0} + \mathcal{D}_{3/0} \rightarrow \overrightarrow{\delta_{O,\,E_2/0}} = \overrightarrow{\delta_{O,\,1/0}} + \overrightarrow{\delta_{O,\,2/0}} + \overrightarrow{\delta_{O,\,3/0}}$$

Calculé question 7

$$\overrightarrow{\delta_{O,\,2/0}} = \overrightarrow{\delta_{I,\,2/0}} + \overrightarrow{OI} \wedge \overrightarrow{R_{d\ 2/0}} \qquad \overrightarrow{\delta_{O,\,3/0}} = \overrightarrow{\delta_{I,\,3/0}} + \overrightarrow{OI} \wedge \overrightarrow{R_{d\ 3/0}}$$

$\overrightarrow{\delta_{I,\,2/0}} = \left.\dfrac{d}{dt}\overrightarrow{\sigma_{I,2/0}}\right|_{B0}$ car I Gdg de S_2

$\overrightarrow{\delta_{I,\,3/0}} = \left.\dfrac{d}{dt}\overrightarrow{\sigma_{I,3/0}}\right|_{B0}$ car I Cdg de S_3

$\overrightarrow{\sigma_{I,\,2/0}}$ calculé question 4

$\overrightarrow{\sigma_{I,\,3/0}}$ calculé question 5